Elementary Integration

1. Evaluate:

(a) $\int \frac{x}{1-2x}dx$ **(b)** $\int \frac{1-x}{2x-1}dx$ **(c)** $\int \frac{3-2x}{2x-1}dx$ **(d)** $\int \frac{x^2}{px-q}dx$

(e) $\int \frac{ax+b}{cx+d}dx$ **(f)** $\int (2+3x)^2 dx$ **(g)** $\int \sqrt[3]{3x+1}dx$ **(h)** $\int x\sqrt{x^2+3}\ dx$

(i) $\int \frac{5}{\sqrt{2x+3}}dx$ **(j)** $\int \frac{t}{\sqrt{1-2t^2}}dt$ **(k)** $\int x^3\sqrt{5+2x^2}dx$ **(l)** $\int x^n\sqrt{qx^{n+1}+p}\ dx$

(m) $\int \frac{x+1}{\sqrt{(x+1)^2+4}}dx$ **(n)** $\int \frac{2t-3}{\sqrt{2t+1}}dt$ **(o)** $\int \frac{x^2}{x^2-4}dx$ **(p)** $\int (2x-3)^{10}dx$

(q) $\int \tan^{10} x\sec^2 x dx$ **(r)** $\int x(\alpha x^2+\beta)^n dx$, n ≠ -1 **(s)** $\int \frac{\sqrt{x^2-4}}{x}dx$ **(t)** $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

(u) $\int_0^{1/\sqrt{5}} x^3(1-5x^2)^{10}dx$ **(v)** $\int_{-1/5}^{1/5} x\sqrt{2-5x^2}dx$ **(w)** $\int_0^{\pi/2}\sin mx\cos mx\ dx$, m ∈ **Z**

2. Evaluate **(a)** $\int_{-1}^{1}|x|dx$ **(b)** $\int_0^2 |1-x|dx$.

3. Find **(a)** $\frac{d}{dx}\int_a^h \sin(x^2)dx$ **(b)** $\frac{d}{da}\int_a^h \sin(x^2)dx$ **(c)** $\frac{d}{dh}\int_a^h \sin(x^2)dx$

4. Evaluate **(a)** $\int_1^2 \frac{(x+1)(x^2-3)}{3x^2}dx$ **(b)** $\int_1^8 \sqrt[3]{x}\ dx$

5. Show that if f is continuous on [a, b] , then $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|dx$.

6. Find **(a)** $\frac{d}{dx}\int_0^{x^2}\sqrt{1+t^2}dt$ **(b)** $\frac{d}{dx}\int_{x^2}^{x^3}\frac{dt}{\sqrt{1+t^4}}$ **(c)** $\frac{d}{dx}\int_{\sin x}^{\cos x}\cos(\pi t^2)dt$.

7. **(a)** By intermediate value theorem, show that if $g(x) > 0$ in [a , b]

$$\int_a^b f(x)g(x)dx = f(x_0)\int_a^b g(x)dx \quad \text{for some } x_0 \in [a, b].$$

Hence, or otherwise, calculate $\lim_{n\to\infty}\int_0^1 \frac{x^n}{1+x}dx$.

(b) By using $x^n = (1+x)x^{n-1} - x^{n-1}$, show that $\int_0^1 \frac{x^n}{1+x}dx = \frac{1}{n} - \int_0^1 \frac{x^{n-1}}{1+x}dx$.

(c) Hence, by using **(a)** and **(b)** , find $\lim_{n\to\infty}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots+(-1)^{n-1}\frac{1}{n}\right]$.